Autonomous Organizational Modes and the Limits of Hiring Flexibility

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1 Abstract

We study workforce evolution in hierarchical organizations as a discrete-time linear dynamical system [[1]; [2]] driven by internal transitions and constrained hiring inputs.
Some workforce profiles can be sustained indefinitely through hiring. Some workforce imbalances can be corrected rapidly, however there are fundamentally different structural questions.
Let

$$x(t+1) = Px(t) + Bu(t) \tag{1.1}$$

where: P describes internal promotion, retention, and attrition, B specifies hiring-entry structure, $u(t)$ represents external hiring inputs.
Sustainable workforce configurations arise from the operator

$$(I - P)^{-1}B \tag{1.2}$$

while long-run adaptation speed is governed by the spectral structure of the uncontrollable component of the dynamics.
The paper develops a decomposition of workforce imbalance into: a reachable component influenced by hiring, and an autonomous component evolving entirely through internal mobility.
That decomposition yields a structural distinction between feasibility and adaptability. An organization may sustain a large collection of workforce profiles while remaining intrinsically slow to reconfigure.

The paper derives finite-horizon minimum-energy hiring laws[[3]; [4]], asymptotic upper and lower decay bounds, and a structural construction showing that organizations with nearly identical hiring flexibility may exhibit different adaptation half-lives due entirely to internal mobility geometry.
The results show that increasing hiring intensity alone cannot overcome structural limits imposed by hierarchical transition pathways.

2 Introduction

Workforce planning is usually framed as an allocation problem[[5]]: given a staffing target and hiring constraints, determine a hiring policy that moves the organization toward that target.
That viewpoint treats hiring as a direct mechanism for adjusting workforce composition, however hierarchical organizations evolve differently.
Hiring typically enters only at a limited collection of levels, while subsequent evolution propagates through promotion, retention, and attrition[[6]]. Workforce adjustments therefore move indirectly through organizational pathways determined by internal But, which workforce imbalances can hiring correct, and which evolve primarily through internal mobility dynamics?
We study this using a linear discrete-time model[[1]; [7]].
The purpose is structural isolation rather than predictive realism. The present work is not intended to introduce new controllability machinery. It develops an organizational interpretation of classical linear systems concepts and isolates structural quantities governing workforce adaptability. The emphasis is therefore conceptual and analytical rather than predictive. Linear time-invariant dynamics allow the interaction between hiring structure and internal mobility to be analyzed analytically before introducing nonlinear or stochastic effects.
We observe three main conclusions.

- sustainable workforce configurations form a linear subspace determined jointly by hiring structure and internal transitions.
- workforce imbalance decomposes into: a component influenced by hiring, and a complementary component evolving autonomously.
- long-run adaptation speed depends on the internal transition operator acting on the uncontrollable component of the dynamics rather than on hiring magnitude itself.

The paper develops a mathematical distinction between sustainability and adaptability and shows that those properties are governed by fundamentally different operators.
Classical controllability theory[[2]; [4]] already contains the mathematical ingredients individually. The workforce setting becomes nontrivial because intervention enters only through limited hiring channels while restructuring propagates indirectly through hierarchical promotion pathways.
The central claim is therefore simple:

Hiring capacity determines which workforce profiles can be sustained and internal mobility geometry determines how rapidly those profiles can be reached.

3 Related Work

3.1 Markov Manpower Models

Bartholomew (1982) formalized organizational mobility as probabilistic flows between discrete grades, establishing the Markov manpower system as the field's canonical model.
Vassiliou (1982) studied asymptotic behavior under time-varying transition matrices. Vassiliou and Tsantas (1984) introduced stochastic recruitment control. Georgiou and Vassiliou (1997) incorporated cost structure into manpower systems. Most of that literature studies maintainability: which workforce distributions can persist under some admissible policy. Far less attention is devoted to structural adaptation limits and transient restructuring dynamics.

3.2 Optimization-Based Planning

Grinold (1976), Dimitriou et al. (2013), and De Feyter and Guerry (2017) formulate workforce planning as constrained optimization over hiring and promotion schedules. Those approaches emphasize feasibility and optimization efficiency, but do not explicitly isolate structural adaptation timescales induced by internal organizational mobility.

3.3 Linear Systems Perspective

The reachable-subspace construction, controllability decomposition, and finite-horizon controllability Gramian used throughout this paper originate from classical linear systems theory [[1]; [2]].
The spectral radius estimates, Jordan asymptotics, and induced norm inequalities rely on standard results from matrix analysis [[8]]. Recent work in network controllability and structured systems theory further emphasizes how system topology constrains control authority in large interconnected systems [[9]; [10]; [11]]. The contribution of the present paper is therefore not new controllability machinery itself.
Rather, the contribution lies in identifying a structural separation between sustainability and adaptability in hierarchical organizations and showing that those quantities can diverge dramatically even under similar hiring flexibility.

3.4 Network and Structured Controllability

Recent work in network controllability studies how system topology constrains control authority in large interconnected systems. Structural controllability theory analyzes controllability properties induced by sparsity patterns rather than precise

numerical parameters.
The present paper differs in emphasis by focusing on hierarchical workforce dynamics in which intervention enters only through restricted hiring channels while restructuring propagates through internal promotion pathways. The resulting adaptation behavior depends strongly on slow autonomous modes associated with upper organizational tiers.

4 Motivating Example

Consider a four-level engineering organization consisting of: junior engineers, mid-level engineers, senior engineers, management.
Suppose external hiring enters through junior and mid-level engineering roles.
Once employees enter the organization, workforce movement occurs internally through promotion and retention.
An increase in junior hiring therefore influences management only indirectly:
hiring changes junior staffing immediately, promotions gradually propagate that change upward, attrition simultaneously removes workers from the system.
Some workforce imbalances can therefore be corrected efficiently. Others decay only gradually because the organization propagates information slowly through promotion pathways.

Junior → Mid → Senior → Management

$$P$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$q_1 \quad q_2 \quad q_3 \quad q_4$$

Figure 1: Hierarchical workforce system with promotion propagation through P and attrition leakage at each organizational tier.

5 Preliminaries

Let

$$x(t) \in \mathbb{R}^n \tag{5.1}$$

represent the workforce distribution across n organizational levels.
The spectral radius of a matrix A is defined by

$$\rho(A) = \max\{|\lambda| : \lambda \in \sigma(A)\} \tag{5.2}$$

[[8]]
A subspace $V \subseteq \mathbb{R}^n$ is invariant under A if[[12]]

$$AV \subseteq V \tag{5.3}$$

The reachable subspace generated by (P, B) is

$$\mathcal{R} = \text{span}\{B, PB, P^2B, ..., P^{n-1}B\} \tag{5.4}$$

[[1]; [2]].
Derivation:
Repeated application of the workforce dynamics shows that hiring interventions generate vectors of the form

$$P^k Bu(0) \tag{5.5}$$

for $k \geq 0$. The reachable subspace therefore consists precisely of imbalance directions generated through hiring followed by internal workforce propagation.

6 Model Formulation

Let workforce evolution satisfy

$$x(t+1) = Px(t) + Bu(t) \tag{6.1}$$

where:
P is the internal transition matrix, B determines hiring-entry locations, $u(t)$ is the hiring vector.
Assume each column of P satisfies

$$\sum_i P_{ij} = 1 - q_j < 1 \tag{6.2}$$

with

$$q_j > 0 \tag{6.3}$$

q_j represents attrition from organizational level j.

6.0.1 Theorem

If every column sum of P is strictly less than 1, then

$$\rho(P) < 1 \tag{6.4}$$

6.0.2 Proof

The induced matrix 1-norm satisfies

$$\|P\|_1 = \max_j \sum_i |P(ij)| \tag{6.5}$$

All entries are nonnegative, so

$$\|P\|_1 = \max_j \sum_i P(ij) \tag{6.6}$$

Using the substochastic assumption[[13]] gives

$$\|P\|_1 = \max_j (1 - q_j) < 1 \tag{6.7}$$

The spectral radius satisfies

$$\rho(P) \leq \|P\|_1 \tag{6.8}$$

[[8]]
Hence

$$\rho(P) < 1 \tag{6.9}$$

Suppose $(I - P)$ were singular. Then 1 would be an eigenvalue of P, contradicting

$$\rho(P) < 1 \tag{6.10}$$

Hence $(I - P)$ is invertible.

7 Sustainable Workforce Configurations

Suppose hiring remains constant:

$$u(t) = u^* \tag{7.1}$$

An equilibrium workforce configuration satisfies

$$x^* = Px^* + Bu^* \tag{7.2}$$

Rearranging gives

$$(I - P)x^* = Bu^* \tag{7.3}$$

Since $(I - P)$ is invertible,

$$x^* = (I - P)^{-1}Bu^* \tag{7.4}$$

Define

$$W = (I - P)^{-1}B \tag{7.5}$$

Then every sustainable workforce configuration has the form

$$x^* = Wu^* \tag{7.6}$$

7.0.1 Proposition

A target workforce configuration x_t is sustainable if and only if

$$x_t \in \text{range}(W) \tag{7.7}$$

Proof:
Since $(I - P)$ is invertible by Theorem 6.0.1, the equilibrium equation

$$(I - P)x_t = Bu^* \tag{7.8}$$

admits a solution if and only if there exists a hiring vector u^* satisfying

$$x_t = (I - P)^{-1}Bu^* = Wu^* \tag{7.9}$$

Hence x_t is sustainable precisely when

$$x_t \in \text{range}(W) \tag{7.10}$$

7.0.2 Interpretation

Feasibility depends jointly on hiring structure and internal workforce propagation. Increasing hiring magnitude alone cannot create missing structural directions.

8 Dynamics Around Equilibrium

Fix an equilibrium pair (x^*, u^*).
Define deviation variables

$$e(t) = x(t) - x^* \tag{8.1}$$

and

$$v(t) = u(t) - u^* \tag{8.2}$$

Substituting into the original dynamics gives

$$x(t+1) - x^* = P(x(t) - x^*) + B(u(t) - u^*) \tag{8.3}$$

hence

$$e(t+1) = Pe(t) + Bv(t) \tag{8.4}$$

[[1]; [14]].
The equilibrium equation determines which workforce configurations are sustainable and the deviation equation determines how rapidly imbalance evolves.

9 Reachability and Decomposition

Define the reachable subspace

$$\mathcal{R} = \text{span}\{B, PB, P^2B, ..., P^{n-1}B\} \tag{9.1}$$

9.0.1 Theorem

The reachable subspace $\mathcal{R}$ is invariant under P[[1]; [2]].

9.0.2 Proof

Take any vector $r \in \mathcal{R}$.
By definition,

$$r = \sum_{k=0}^{n-1} P^k B a_k \tag{9.2}$$

Applying P gives

$$Pr = \sum_{k=0}^{n-1} P^{k+1} B a_k \tag{9.3}$$

The Cayley-Hamilton theorem implies that P^n may be written as a linear combination of lower powers of P. [[2]]
Hence $P^n B \in \mathcal{R}$.
Therefore every term in Pr lies in $\mathcal{R}$.
Thus

$$P\mathcal{R} \subseteq \mathcal{R} \tag{9.4}$$

9.0.3 Corollary

There exists a complementary subspace U satisfying

$$\mathbb{R}^n = \mathcal{R} \oplus U \tag{9.5}$$

Every imbalance vector therefore decomposes uniquely as

$$e(0) = e_{c(0)} + e_{u(0)} \tag{9.6}$$

with

$$e_{c(0)} \in \mathcal{R} \tag{9.7}$$

and

$$e_{u(0)} \in U \tag{9.8}$$

Workforce Imbalance

$\downarrow$

$\mathcal{R}$ U

Reachable Autonomous

via Hiring Internal Modes

Fast Correction Slow Structural Decay

Figure 2: Decomposition of workforce imbalance into hiring-reachable and autonomous organizational modes.

9.0.4 Interpretation

The controllable component evolves in directions influenced by hiring and the uncontrollable component evolves through internal mobility alone.

10 Minimum Hiring Effort

Reachability determines whether imbalance can be influenced at all. Now we look at how difficult that influence becomes quantitatively.
Define the finite-horizon controllability Gramian

$$W_T = \sum_{k=0}^{T-1} P^k B B^T \left(P^k\right)^T \tag{10.1}$$

The finite-horizon controllability Gramian is a classical object in linear systems theory and characterizes the minimum energy required to drive reachable states between configurations [[1]; [2]].

10.0.1 Theorem

Assume W_T is positive definite on $\mathcal{R}$.
Then the minimum input energy required to drive

$$e_{c(T)} = 0 \tag{10.2}$$

equals

$$\min \sum_{t=0}^{T-1} \|v(t)\|^2 = \left(P^T e_{c(0)}\right)^T W_T^{-1} \left(P^T e_{c(0)}\right) \tag{10.3}$$

[[3]; [4]]

11 Structural Limits on Adaptation

Let

$$P_u = P|_U \tag{11.1}$$

be the restriction of P to the uncontrollable subspace.
Define

$$\lambda_* = \rho(P_u) \tag{11.2}$$

[[8]; [12]]. λ_* measures the asymptotic decay rate of autonomous imbalance modes.

11.0.1 Theorem (Upper Bound)

There exist constants $C > 0$ and $k \geq 0$ such that

$$\|e_{u(t)}\| \leq Ct^k (\lambda_*)^t \tag{11.3}$$

[[8]]

11.0.2 Theorem (Lower Bound)

Suppose $e_{u(0)}$ has nonzero projection onto a dominant Jordan mode of P_u.
Then there exists $c > 0$ such that

$$\|e_{u(t)}\| \geq ct^k (\lambda_*)^t \tag{11.4}$$

for sufficiently large t.

11.0.3 Proof

Restrict the autonomous dynamics to the invariant subspace U and write

$$P_u = MJM^{-1} \tag{11.5}$$

where J is the Jordan canonical form of P_u.
Let $\lambda_* = \rho(P_u)$ denote the spectral radius of P_u, and suppose the largest Jordan block associated with an eigenvalue satisfying $|\lambda| = \lambda_*$ has size $(k+1) \times (k+1)$. Then the corresponding Jordan block may be written as

$$J_* = \lambda_*(I + N) \tag{11.6}$$

where N is nilpotent with $N^{k+1} = 0$.
Using the binomial expansion,

$$J_*^t = \lambda_*^t \sum_{j=0}^{k} \binom{t}{j} N^j \lambda_*^{-j}. \tag{11.7}$$

The dominant contribution therefore satisfies

$$J_*^t = \Theta\big(t^k \lambda_*^t\big). \tag{11.8}$$

If the initial autonomous projection $M^{-1}e_{u(0)}$ possesses a nonzero component along the highest-order generalized eigenvector associated with this block, then there exists a constant $c > 0$ such that

$$\|e_{u(t)}\| \geq c t^k \lambda_*^t \tag{11.9}$$

for all sufficiently large t.

Thus the asymptotic lower bound is governed jointly by the dominant spectral radius and maximal Jordan-chain length.

11.1 Robustness under weak coupling

The previous asymptotic estimates persist under sufficiently small perturbations of the uncontrollable dynamics. Suppose

$$P_\varepsilon = P_0 + \varepsilon \Delta P \tag{11.10}$$

where P_0 is an uncontrollable restriction P_u with dominant spectral radius λ_*.
If ε is sufficiently small, classical matrix perturbation theory implies that the dominant eigenvalues of the perturbed restriction vary continuously with ε.
Consequently, organizations possessing weak coupling between nominally uncontrollable tiers and hiring-accessible tiers may still exhibit adaptation timescales arbitrarily close to those predicted by the uncoupled model.

11.1.1 Proposition

Let $P_{u,\varepsilon}$ denote the perturbed uncontrollable restriction satisfying

$$\|P_{u,\varepsilon} - P_u\| < \varepsilon \tag{11.11}$$

Then for sufficiently small ε,

$$|\rho(P_{u,\varepsilon}) - \rho(P_u)| < C\varepsilon \tag{11.12}$$

for some constant $C > 0$. Hence slow autonomous restructuring modes remain structurally persistent under sufficiently small perturbations of the promotion topology.

12 Structural Adaptability Paradox

Increasing hiring flexibility does not necessarily improve long-run adaptation speed.

12.0.1 Proposition

Consider two organizations with transition operators P_1 and P_2 sharing the same uncontrollable restriction:

$$P_1|_U = P_2|_U \tag{12.1}$$

Suppose the two organizations possess different hiring reachability but identical autonomous restrictions.

$$\mathcal{R}_1 \subsetneq \mathcal{R}_2 \tag{12.2}$$

Then Organization 2 can sustain strictly more workforce configurations, while both organizations possess identical asymptotic adaptation rates.

12.0.2 Interpretation

Hiring flexibility and adaptation speed are different organizational resources.
An organization may dramatically expand the collection of workforce profiles it can sustain while remaining equally slow to restructure.

13 Structural Construction

We now construct two organizations possessing nearly identical hiring flexibility while exhibiting dramatically different adaptation half-lives.

13.1 Organizational Structure

Consider again a four-level engineering organization consisting of: junior engineers, mid-level engineers, senior engineers, management.
Suppose external hiring enters through junior engineering and mid-level engineering roles.
The hiring structure becomes

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{13.1}$$

A key structural feature of both organizations is that management receives no direct promotion inflow from senior engineers:

$$P_{4,3} = 0 \tag{13.2}$$

in both transition matrices.
Management evolves entirely through its own retention dynamics.
Consequently, regardless of hiring volume, the management level lies outside the reachable subspace.
The uncontrollable subspace is

$$U = \text{span}\{e_4\} \tag{13.3}$$

and the restriction P_u is the scalar

$$P_{4,4} \tag{13.4}$$

which is the management retention rate.
The two organizations below differ only in internal mobility structure.
The assumption $P_{4,3} = 0$ is intentionally chosen as a mathematically clean boundary case that isolates the autonomous restructuring mode exactly.
The purpose is analytical transparency rather than institutional realism. The construction should therefore be interpreted as a minimal structural example rather than a calibrated organizational model.
Its purpose is isolating the interaction between hiring reachability and autonomous restructuring modes in the clearest possible setting.
The same qualitative divergence in adaptation half-lives persists under substantially weaker assumptions.

Even when $P_{4,3} > 0$, identical structural phenomena arise whenever the hiring operator B lacks sufficient directional authority to actively dominate the slow internal mixing timescales associated with upper organizational tiers.

13.2 Organization A: Slow Adaptation

Suppose promotion probabilities are weak and managerial retention[[6]] is extremely high.
The transition matrix becomes(example)

$$P_A = \begin{pmatrix} 0.80 & 0.15 & 0.00 & 0.00 \\ 0.08 & 0.78 & 0.10 & 0.00 \\ 0.00 & 0.08 & 0.82 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.90 \end{pmatrix} \tag{13.5}$$

The equilibrium feasibility operator equals

$$W_A = (I - P_A)^{-1}B \tag{13.6}$$

Direct computation gives

$$\text{rank}(W_A) = 2 \tag{13.7}$$

with

$$W_A = (I - P_A)^{-1}B = \begin{pmatrix} 7.60 & 6.49 \\ 3.61 & 0 \\ 3.46 & 8.65 \\ 4.81 & 0 \end{pmatrix} \tag{13.8}$$

The zero bottom row confirms management is unreachable through any constant hiring policy. Hence the uncontrollable subspace is one-dimensional.
Since $P_{4,3} = 0$, no hiring intervention, however propagated through promotion can affect the management coordinate.
The uncontrollable subspace is therefore

$$U = \text{span}\{e_4\} \tag{13.9}$$

The restriction of P_A to U is the scalar

$$P_U^A = P_{4,4} = 0.90 \tag{13.10}$$

Hence

$$\rho\big(P_U^A\big) = 0.90 \tag{13.11}$$

The uncontrollable imbalance therefore decays $\approx (0.90)^t$
The corresponding adaptation half-life satisfies

$$(0.90)^t = \frac{1}{2} \tag{13.12}$$

which gives

$$t \approx 6.58 \tag{13.13}$$

organizational cycles.

13.3 Organization B: Fast Adaptation

Suppose promotion pathways become substantially more active and managerial turnover increases.
The transition matrix becomes (example)

$$P_B = \begin{pmatrix} 0.62 & 0.24 & 0.00 & 0.00 \\ 0.18 & 0.58 & 0.20 & 0.00 \\ 0.00 & 0.18 & 0.55 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.52 \end{pmatrix} \tag{13.14}$$

The hiring structure remains identical.
Now,

$$\operatorname{rank}(W_B) = 2 \tag{13.15}$$

with

$$W_B = (I - P_B)^{-1} B = \begin{pmatrix} 3.95 & 2.79 \\ 1.12 & 0 \\ 2.09 & 4.42 \\ 1.77 & 0 \end{pmatrix} \tag{13.16}$$

By the same structural argument,

$$U = \operatorname{span}\{e_4\} \tag{13.17}$$

and

$$P_U^B = P_{4,4} = 0.52 \tag{13.18}$$

Hence

$$\rho\left(P_U^B\right) = 0.52 \tag{13.19}$$

The uncontrollable imbalance therefore decays $\approx (0.52)^t$
The corresponding adaptation half-life satisfies

$$(0.52)^t = \frac{1}{2} \tag{13.20}$$

which gives

$$t \approx 1.06 \tag{13.21}$$

organizational cycles.

13.4 Comparison

Organization	rank(W)	$\rho(P_u)$	Adaptation Half-Life
A	2	0.90	6.58 cycles
B	2	0.52	1.06 cycles

13.4.1 Autonomous Decay Comparison

Suppose the autonomous restrictions satisfy

$$\rho\left(P_{u,A}\right) = 0.90 \tag{13.22}$$

and

$$\rho\left(P_{u,B}\right) = 0.52 \tag{13.23}$$

respectively. The autonomous restructuring magnitudes therefore decay asymptotically like

$$\|e_{u,A}(t)\| \approx C_A(0.90)^t \tag{13.24}$$

and

$$\|e_{u,B}(t)\| \approx C_B(0.52)^t \tag{13.25}$$

for sufficiently large t. After 20 organizational cycles,

$$(0.90)^{20} \approx 0.1216 \tag{13.26}$$

while

$$(0.52)^{20} \approx 2.09 \times 10^{-6} \tag{13.27}$$

Thus Organization B eliminates autonomous imbalance several orders of magnitude faster despite comparable hiring reachability.

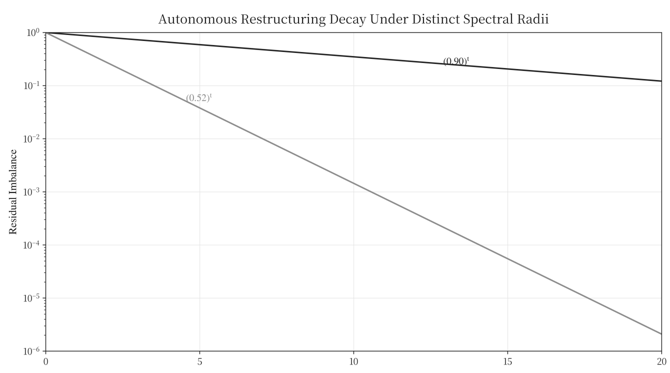

Figure 3: Autonomous restructuring decay under distinct spectral radii.

13.5 Interpretation

The two organizations possess comparable equilibrium feasibility yet dramatically different restructuring timescales. The difference emerges entirely from the autonomous spectral structure of the uncontrollable dynamics. In Organization A, the high management retention rate creates a persistent autonomous mode that

decays slowly regardless of hiring intensity.
In Organization B, increased managerial turnover substantially accelerates autonomous decay. Thus organizational adaptability depends fundamentally on internal mobility geometry rather than hiring flexibility alone.

14 Toward Organizational Design

The previous construction suggests a broader organizational design problem. Given fixed hiring-entry structure, the objective becomes minimizing the dominant autonomous spectral radius

$$\rho(P_u). \tag{14.1}$$

subject to operational and organizational constraints.

15 Discussion

The analysis separates equilibrium feasibility from intrinsic restructuring speed. Hiring flexibility determines which workforce profiles may be sustained, while the autonomous spectral structure governs long-run adaptation timescales. The dominant eigenmodes of the uncontrollable dynamics therefore provide a quantitative measure of organizational rigidity.

15.1 Limitations

The present framework intentionally abstracts away several important workforce features including nonlinear promotion effects, heterogeneous employee quality, strategic behavior, endogenous attrition, and stochastic hiring constraints.
The model should therefore be interpreted as a structural dynamical framework rather than a predictive organizational simulator. In particular, the analysis isolates how internal transition geometry alone can produce persistent restructuring bottlenecks even under substantial hiring flexibility.

16 Future Research Directions

Some ways I see this going forward is:-

16.1 Nonlinear Organizational Dynamics

The present model assumes linear workforce propagation. Replacing P with a state-dependent operator $P(x)$ would produce nonlinear organizational dynamics with potentially multiple equilibria, congestion effects, and instability induced by promotion bottlenecks.

16.2 Stochastic Workforce Evolution

Random attrition, uncertain hiring conditions, and fluctuating promotion rates may be incorporated through stochastic propagation operators. Such extensions would connect the framework to Markov workforce systems and probabilistic organizational resilience analysis.

16.3 Adaptive Hiring Policies

The current analysis assumes static hiring structure through a fixed operator B. Allowing adaptive or state-dependent hiring policies would lead naturally to feedback-control formulations in which hiring intensity responds dynamically to workforce imbalance.

16.4 Empirical Organizational Estimation

The operators P and B may in principle be estimated from longitudinal workforce data. This would permit empirical measurement of organizational restructuring half-lives and autonomous adaptation modes in real institutions.

16.5 Organizational Design Optimization

The decomposition between reachable and autonomous restructuring directions suggests a broader organizational design problem: minimizing the dominant autonomous spectral radius

$$\rho(P_u) \tag{16.1}$$

subject to operational and structural constraints. Such optimization problems may provide quantitative measures of institutional rigidity and long-run organizational adaptability.

17 Conclusion

We modeled workforce evolution as a linear dynamical system driven jointly by internal transitions and external hiring.
The analysis separates workforce imbalance into controllable and uncontrollable components and shows that long-run adaptation speed is fundamentally constrained by internal organizational structure.
The paper's central contribution is the identification of a structural separation between sustainability and adaptability in hierarchical organizations.
Organizations possessing similar hiring flexibility may exhibit radically different restructuring times because hiring access and autonomous mobility geometry are governed by different operators.
Broader hiring flexibility therefore does not necessarily imply faster organizational

restructuring. The framework connects workforce planning, controllability theory, spectral analysis, and organizational design within a unified mathematical model.

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